

WHAT DO EXPERIMENTAL DATA "SAY" ABOUT GROWTH OF HADRONIC TOTAL CROSS-SECTIONS ?

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Abstract We reanalyse $\bar{p}p$ and pp high energy data of the elastic scattering above $\sqrt{s} = 5$ GeV on the total cross-section σ_{tot} and on the forward ρ -ratio for various models of Pomeron, utilizing two methods. The first one is based on analytic amplitudes, the other one relies on assumptions for σ_{tot} and on dispersion relation for ρ . We argue that it is **not** possible, from fitting only existing data for forward scattering, to select a definite asymptotic growth with the energy of σ_{tot} . We find equivalent fits to the data together with a logarithmic Pomeron giving a behavior $\sigma_{tot} \propto \ln^\gamma s$, $\gamma \in [0.5, 2.20]$ and with a supercritical Pomeron giving a behavior $\sigma_{tot} \propto s^\epsilon$, $\epsilon \in [0.01, 0.10]$.

1 Introduction

The question "How fast do total cross-sections of hadronic interaction rise?" is discussed permanently (see for example [1] and references therein). A definite answer to this question, correlated to that of the prediction of the asymptotic growth is still lacking. The Tevatron is at the lower edge of the asymptotic range, while the cosmic rays data (because of large uncertainties) may serve only as a guide. Interesting information on the energy dependence of the total cross-sections are expected from future experiments projected near the new generation of colliders, for example at the LHC [2] with 10 to 14 TeV center of mass energies, where, presumably, the asymptotic range will have been reached.

There is a number of theoretical and phenomenological models together with fitting procedures using various selections of data that lead to controversial predictions about an asymptotic growth of the total cross-sections. To avoid an overlong list of papers devoted to this subject, we do not give the references for all original papers. We only note that the main prediction varies from a very slow growth when the Mandelstam variable $s \rightarrow \infty$

$$\sigma_{tot} \propto \ln^\gamma(s/s_0), \quad \gamma \simeq 0.4 \quad (1)$$

as for the "critical" Pomeron in the Reggeon Field Theory [3], up to a very fast growth

$$\sigma_{tot}(s) \propto (s/s_0)^\epsilon, \quad \epsilon \simeq 0.1 \quad (2)$$

as in the model of the "supercritical" Pomeron, popular with the work [4] of Donnachie-Landshoff (DL); of course the latter "overunitary" behavior should be changed at very high energies to respect the Froissart-Martin bound [5]

$$\sigma_{tot} \leq \frac{\pi}{m_\pi^2} \ln^2(s/s_0). \quad (3)$$

As a rule, each hypothesis exhibits a good agreement with the experimental data. At the same time, an accurate comparison of the quality of the fits given by different models is not easy because often a single model is discussed in details with a specific set of experimental data. Only a comparison between models that uses a common and complete set of experimental data is able to bring an answer to the question in the title of this note.

Two methods are intensively investigated to analyse the experimental data on the total cross-sections ($\sigma_{tot}(s)$) and on the ratio of the real to the imaginary part of the forward scattering amplitude ($\rho(s)$) :

-1) the first one uses an explicit analytic form of the complex scattering amplitude $A(s, t = 0)$

-2) the other method is based on phenomenological assumptions on the total cross-section $\sigma_{tot}(s)$ complemented by the dispersion relation between the real and imaginary part of the forward elastic amplitude. Because a nonvanishing cross-section is assumed with increasing energy, a dispersion relation with one subtraction constant is used.

In both methods, the free parameters are determined by a fit to the experimental data. *A priori* they should lead to similar (if not identical) conclusions for the behavior of $\sigma_{tot}(s)$. Unfortunately, it is not the case. As an example, let us quote first Ref. [6], where the method using analytic amplitude was applied for the analysis of data : "It is found that available experimental data at $t = 0$ do not seem to indicate a growth of the total cross-sections faster than the first power in logarithm of the energy.". Let us quote also a recent paper [7] where the approach using dispersion relation is investigated in the same energy range : "For the whole set of present data statistical analysis ($\chi^2/d.o.f.$) seems to favour a "Froissart-like" (logarithmic $(\ln s)^{\gamma \approx 2}$) rise of the total cross-section rather than a "Regge-like" (power s^ϵ) one". A similar conclusion holds in [8]. Thus, there is a certain contradiction between the conclusions obtained in two approaches of analysing the data. One may find many other recent examples of contradictory conclusions : for example, Block and White [9] are supporters of $\ln s$, while Kang and Kim [10] argue that models with $\ln s$ and $\ln^2 s$ are equally compatible with the data; Bertini *et al.* [11] from an analytic evaluation of the real part of the forward scattering amplitude, based on the dispersion relations, report equivalent success for $\ln^2 s$ and s^ϵ behaviors; Cudell *et al.* [12] exhibit the success of a DL-like model....

This note is a tentative to clear up this problem. For that purpose, we analyse the current experimental data by the above mentioned methods using a common set of data. We include in our analysis all published data [13], excluding only those at low and intermediate energies with an error greater than 1 mb and those which were corrected by experimentalists in a later publication. Finally, we are left with a grand total of 187 non filtered data distributed as follows : for total cross-sections of pp (72 points), $\bar{p}p$

(47 points)-scattering and for the ratio (ρ) of real to imaginary part of the forward amplitude (correspondingly 55 and 13 points) at the energies

$$5 \text{ GeV} \leq \sqrt{s} \leq 1800 \text{ GeV}.$$

Of course, such a set favors low energy data (especially pp), but from a theoretician point of view it seems to us hazardous to omit some experimental data. However, to satisfy a suspicious reader, unwilling to manage with the discrepancies at the Tevatron between CDF and E710 results, we also discuss the consequences of excluding the (five) values above 546 GeV from the data set.

To be complete, we explored formulations giving two alternatives for the asymptotic behavior of $\sigma_{tot}(s)$: logarithmic (as in (1)) and power-like (as in (2)) rising with energy.

2 "Analytic amplitude" and "dispersion relation" fits

-1) Analytic amplitude fit.

The (Born-level) amplitude for elastic forward scattering is the explicit analytic function of the center of mass energy \sqrt{s}

$$A^\pm(s, 0) = P(s) + F(s) \pm \Omega(s), \quad (4)$$

where $+$ ($-$) stands for $\bar{p}p$ (pp) respectively and where, as usual in the Regge approach, the secondary (f - and ω -) Reggeon contributions are

$$F(s) = ig_f(-is/s_0)^{\alpha_f(0)-1}, \quad (5)$$

$$\Omega(s) = g_\omega(-is/s_0)^{\alpha_\omega(0)-1}. \quad (6)$$

For the Pomeron contribution, we consider two parametrizations. The first one is a standard picture for a logarithmic Pomeron, (with a unit intercept, leading to a logarithmic rising asymptotic total cross-section)

$$P(s) = i[g_0 + g_1 \ln^\gamma(-is/s_0)]. \quad (7)$$

The second one is a generalization of the DL model [4] for the supercritical Pomeron (with an intercept $\alpha_p(0) = 1 + \epsilon$, leading to a power rising asymptotic total cross-section)

$$P(s) = i[g_0 + g_1(-is/s_0)^\epsilon], \quad \epsilon > 0, \quad (8)$$

where the constant term we add stands for a preasymptotic contribution. We have also investigated the original DL parametrization with identical Reggeon intercepts $\alpha_f(0) = \alpha_\omega(0)$ and a simplified Pomeron $g_0 = 0$. We concluded as in [6, 7] that this option, leading to a too high χ^2 , is not suitable in the present energy range and should be rejected on behalf of the general case $g_0 \neq 0$, $\alpha_f(0) \neq \alpha_\omega(0)$.

We satisfy the optical theorem (normalization) with

$$\sigma_{tot} = \sigma^\pm(s) = 8\pi \Im A^\pm(s, 0) \quad (9)$$

and set everywhere $s_0 = 1 \text{ GeV}^2$.

The calculation of the ρ -ratio is straightforward from its definition

$$\rho^\pm(s) = \frac{\Re A^\pm(s, 0)}{\Im A^\pm(s, 0)} . \quad (10)$$

Within this formulation, we are left with 7 free parameters (4 "coupling" constants, 3 intercepts –or power–) to be determined by a χ^2 minimisation (from the fit to the experimental data on σ^\pm and ρ^\pm). We note that the CERN standard computer program for χ^2 minimization "MINUIT" does not "feel" a sensitive dependence of amplitudes with respect to γ in (7) –and ϵ in (8)–. Actually, for all investigated cases, we found an output value of this parameter very closed to the input one. Therefore, we adopt a procedure which consists in fitting the other 6 free parameters for fixed values of γ –or ϵ – and then plotting χ^2 versus γ –and ϵ – to deduce the optimal value. This is time-consuming, mainly because it is not straightforward to find the minimum χ^2 for a given γ –or ϵ –, but in our opinion, it is the price to pay to extract a valuable conclusion.

-2) Dispersion relation fit.

In this method the total cross-sections σ^\pm are parametrized explicitly. The phenomenological expressions for σ^\pm are defined in Refs.[7, 8, 14] as follows (we use the same notations for the relevant terms as in the analytic form of amplitudes in order to emphasize the similarities and to permit a transparent comparison of results)

$$\sigma^\pm = \sigma_P + \sigma_f \pm \sigma_\omega, \quad (11)$$

with

$$\sigma_f = g_f(E/E_0)^{\alpha_f(0)-1}, \quad (12)$$

$$\sigma_\omega = g_\omega (E/E_0)^{\alpha_\omega(0)-1} \quad (13)$$

and first of all

$$\sigma_P = g_0 + g_1 \ln^\gamma(s/s_0). \quad (14)$$

In relation with the expected smallness of ϵ , we also consider the supercritical Pomeron model (generalized DL form)

$$\sigma_P = g_0 + g_1 (s/s_0)^\epsilon. \quad (15)$$

The ratios ρ^\pm of the real to imaginary part of amplitudes are calculated from the dispersion relation [15]

$$\rho^\pm(s) \sigma^\pm(s) = \frac{B}{p} + \frac{E}{\pi p} \int_m^\infty dE' p' \left[\frac{\sigma^\pm(s')}{E'(E' - E)} - \frac{\sigma^\mp(s')}{E'(E' + E)} \right]. \quad (16)$$

In these equations $E(p)$ is the proton energy (3-momentum) in the laboratory system, $E_0 = 1$ GeV, m is the proton mass and B is a subtraction constant. Thus, the real part of the amplitudes depends on the asymptotic cross-sections.

We note that, for historical reasons of convenience, the contributions of the secondary Reggeons and of the Pomeron include different variables (namely E for the Reggeons in (12,13), s for the Pomeron in (14,15) and $E_0 = 1$ GeV does not correspond to $s_0 = 1$ GeV²). We call this mixed formulation a "ES-parametrization". In order to perform a complete comparison of various models, in addition we also considered more coherently that we call a "SS-parametrization" and a "EE-parametrization". In the SS-parametrization, the variable E/E_0 in (12) and (13) is changed for s/s_0 . In the EE-parametrization, the variable s/s_0 as well as E/E_0 are changed for E/m (it is necessary to use m instead of $E_0 = 1$ GeV because $\ln E/E_0 \leq 0$ for $m \leq E \leq E_0$ implies σ_P complex for $\gamma \neq 1$ or 2).

Within this second formulation, we have 8 free parameters (4 "cross-section" constants, 3 intercepts –or power– and the subtraction constant) to be determined by a χ^2 minimisation. The same remark as above for the determination of γ –and ϵ – holds, consequently we adopt a similar point of view : minimizing over the other 7 free parameters for fixed values of γ –or ϵ – and then deducing the optimal γ –and ϵ – value from the plot χ^2 versus γ –and ϵ –.

3 Results and discussion.

A selection of our results of $\chi_{d.o.f}^2$ versus γ and ϵ (*i.e.* for a logarithmic and a supercritical Pomeron) is displayed in Figs.1a-b for the analytical amplitude fits and in Figs.2a-b for the dispersion relation fits.

We find (see Fig.1) that the $\chi_{d.o.f}^2$ issued from the truncated data set (with $\sqrt{s} \in [5, 546]$ GeV) is slightly better than the $\chi_{d.o.f}^2$ issued from the entire data set (with $\sqrt{s} \in [5, 1800]$ GeV) : 1.10 instead of 1.15, the two sets giving homothetic curves. However, these differences are not significant (the corresponding parameters yield undistinguishable plots of the cross-sections and of the ρ -ratios); they are due essentially to the data in conflict at high energies. This trend is exhibited only in the case of analytical fits but might be extended to dispersion relation fits.

We find also (see Fig.2) that the $\chi_{d.o.f}^2$ are quite comparable in the three (ES, SS, EE) parametrizations.

Furthermore, the examination of Figs.1 and 2 does not allow to find a significant hierarchy

(i) between the logarithmic Pomeron and the supercritical Pomeron

(ii) and, to a lesser extend, between the first method using analytical amplitudes and the second method using phenomenological cross-sections and the dispersion relation; we only note the second one gives a minimal $\chi_{d.o.f}^2 \simeq 1.10 - 1.11$, slightly better than $\chi_{d.o.f}^2 \simeq 1.14 - 1.15$ for the first one. Let us mention also that this second method would give as an indication

$$\gamma \sim 2.2 \text{ and } \epsilon \sim 0.07 .$$

However, all of these figures (in spite of a large vertical scale) display a wide plateau in the χ^2 behavior. So, the proposed values must not be separated from the wideness of this plateau. The first method is somewhat more selective to exclude highest values of γ and ϵ , while the second one excludes lowest values of these parameters. Collecting all our results we finally conclude :

a logarithmic rise in $\ln^\gamma s$, $\gamma \in [0.5, 2.20]$

and a power-like rise in s^ϵ , $\epsilon \in [0.01, 0.10]$

are equally probable. By this, we mean that at the same time the corresponding $\chi_{d.o.f}^2$ are in the range $[1.10 - 1.20]$ and the corresponding plots of $t = 0$

observables give fits which cannot be preferred "by eye". As a consequence, we emphasize that it is impossible to settle any definite preferable values of γ and ϵ , inside the above wide ranges. This is an illustration of an important feature of the asymptotic total cross-section, as it may be predicted from actual data above 5 GeV.

As a by-product, it is interesting to note that the dipole Pomeron model (\equiv logarithmic Pomeron with $\gamma = 1$) and the supercritical Pomeron model with very small ϵ are similar in describing the available experimental data. Indeed, one can approximate, for limited values of s and small ϵ , the series expansion of the supercritical Pomeron contribution (8) as

$$P(s) = i \left[g_0 + g_1 \sum_{n=0}^{\infty} \frac{[\epsilon \ln(-is/s_0)]^n}{n!} \right] \simeq i [g_0 + g_1 [1 + \epsilon \ln(-is/s_0)]] \quad (17)$$

and, redefining the Pomeron coupling constants, identify with the logarithmic Pomeron (7) with $\gamma = 1$ (f -, ω -couplings and intercepts for the two descriptions are unchanged). Consequently, for any very small ϵ ($\epsilon < 0.01$ for the present energies)

$$\chi^2(\text{supercrit. with } \epsilon \rightarrow 0) \longrightarrow \chi^2(\text{log. with } \gamma = 1).$$

These properties are still valid in the SE-, EE- and SS-parametrizations and are checked from the fits and in Figs.1-2. Therefore, one can extend the interval of allowed ϵ down to any small non-zero value.

We confirm two main results of Kang and Kim [10] concerning the γ determination and the failure of the original DL model. The first one supports a possible asymptotic cross-section either in $\ln s$ or $\ln^2 s$. The second one (based on a worse $\chi^2_{d.o.f}$ obtained for DL parametrization) simply alleviates the need for a modification of the original model but does not rule out a s^ϵ -behavior. To that respect, we recall that the minor modification we introduced in the DL parametrization (*i.e.* non identical Reggeon intercepts $\alpha_f(0) \neq \alpha_\omega(0)$ and a Pomeron with a constant term $g_0 \neq 0$) appears to be one solution for improving the fit.

Such a philosophy has been previously adopted by Cudell *et al.* [12], who also modified the original DL model and concluded on a fit competitive with any other good fit.

The Pomeron intercept we proposed above slightly differs from the central value of recent fits in [12] : $\alpha_P(0) = 1 + \epsilon \in [1.07, 1.11]$. This is not surprising

because the authors use a parametrization closer than ours to the original DL model, where [3] $\alpha_P(0) = 1.0808$. The lower energy cut in the data set we choose (namely 5 GeV instead of 10 GeV in [12]) which is, once again, a motivation for our generalization of the DL parametrization is responsible for that situation. The Pomeron intercept is probably underestimated in our fits including only pp and $\bar{p}p$ data concentrated at low energies where the Pomeron contribution is expected to be relatively small. To that respect, one must note following Covolan *et al.* [16] that, including in addition $\pi^\pm p$ and $K^\pm p$ data supports a higher intercept at the Born level 1.104, which is still shifted after eikonalization up to 1.122.

As an additional remark, we note that increasing the number of parameters is not necessary to get a better fit, and therefore there is little place for introducing an Odderon at $t=0$: we do not mean that it does not exist but simply that the $t = 0$ -data are not constraining enough to determine its parameters (similarly, they do not allow to distinguish a - from f -meson and ρ - from ω -meson).

As an example of the analytical fits we obtain, we show σ_{tot} versus \sqrt{s} in Fig.3 and ρ versus \sqrt{s} in Fig.4 for $\gamma = 1$ and $\gamma = 2$ in (7) and for $\epsilon = 0.07$ in (8). The parameters for these particular cases are listed in Table 1. Despite the fact that the weight of the highest energies data are small in the fit, one obtains a very good agreement at all energies. In the energy range experimentally known the three sets of results are very similar : the $\gamma = 2$ logarithmic Pomeron and the $\epsilon = 0.07$ power-like Pomeron give indiscernable curves up to the Tevatron energy, a visible splitting with the $\gamma = 1$ logarithmic Pomeron occurs only above 500 GeV for the cross-sections and above 200 GeV for the ρ -ratios. Predictions of these models for the highest energies are presented in Table 2 and in Fig.5. The three cases exhibited are in agreement with cosmic ray data [17], but the LHC results would allow a selection.

To summarize our main conclusion : it seems impossible, at present, to establish whether a $\ln s$ or a $\ln^2 s$ or an s^ϵ behavior for the total cross sections is favored by the data. All the above rises are equally admissible unless a combined fit including also angular distributions is performed. It is our belief that different authors will agree with each other only when new very high energy experimental results will become available and, hopefully, asymptopia will have been reached.

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| | <i>logarithmic</i> ($\gamma = 2.0$) | <i>logarithmic</i> ($\gamma = 1.0$) | <i>super.</i> ($\epsilon = 0.07$) |
|------------------------------------|--|--|--|
| g_f (mb) | 2.385 | 4.559 | 2.971 |
| $\alpha_f(0)$ (GeV $^{-2}$) | 0.674 | 0.804 | 0.711 |
| g_ω (mb) | 1.851 | 1.831 | 1.854 |
| $\alpha_\omega(0)$ (GeV $^{-2}$) | 0.436 | 0.441 | 0.435 |
| g_0 (mb) | 0.9978 | -1.396 | -1.009 |
| g_1 (mb) | 0.9114 | 0.2762 | 1.412 |
| χ^2_{dof} | 1.150 | 1.181 | 1.174 |

Table 1

Parameters of the analytic amplitude for the f -Reggeon (5), the ω -Reggeon (6) and the Pomeron either logarithmic (7) with fixed $\gamma = 2$, or $\gamma = 1$, or supercritical (8) with fixed $\epsilon = 0.07$. The 3 sets are fitted to the 187 data with $\sqrt{s} \geq 5$ GeV.

| | <i>logarithmic</i> ($\gamma = 2.0$) | <i>logarithmic</i> ($\gamma = 1.0$) | <i>super.</i> ($\epsilon = 0.07$) |
|---------------------------|--|--|--|
| ρ (1.8 TeV) | 0.138 | 0.121 | 0.140(0.143) |
| σ (1.8 TeV) in mb | 76.5 | 74.7 | 76.3 |
| ρ (14 TeV) | 0.126 | 0.101 | 0.135 |
| σ (14 TeV) in mb | 108. | 100. | 109. |

Table2.

High energy pp predictions for the total cross-section σ and ρ -ratio, calculated with the analytic amplitudes and parameters from Table 1. The $\bar{p}p$ results are the same, except the ρ -value in parentheses.

Figure captions

Fig.1

-a) γ -dependence of $\chi_{d.o.f}^2$ for the analytic amplitude fit (4,7), including a logarithmic Pomeron leading to a total cross-section rising asymptotically as $\ln^\gamma s$.

-b) ϵ -dependence of $\chi_{d.o.f}^2$ for the analytic amplitude fit (4,8), including a supercritical Pomeron leading to a total cross-section rising asymptotically as s^ϵ .

The solid lines correspond to fits performed with the non-filtered data set between 5 GeV and 1800 GeV; in dashed lines are the results from similar fits but omitting data above 546 GeV.

Fig.2

-a) γ -dependence of $\chi_{d.o.f}^2$ for the dispersion relation fit and the logarithmic Pomeron (11-14,16).

-b) ϵ -dependence of $\chi_{d.o.f}^2$ for the dispersion relation fit and the supercritical Pomeron (11-13,15-16)

All the fits are performed with the non-filtered data set. The solid lines correspond to fits performed within the ES-parametrization, the dashed lines to SS-parametrization, the dotted lines to EE-parametrization.

Fig.3

Behavior of the cross-sections $\sigma_{pp}(s)$ and $\sigma_{\bar{p}p}(s)$ as given from the analytical fits for $\gamma = 2$ (solid line), for $\gamma = 1$ (dashed line) and for $\epsilon = 0.07$ (dotted line). The other 6 free parameters are listed in Table 1.

Fig.4

Same as in Fig.3 for the ratios $\rho_{pp}(s)$ and $\rho_{\bar{p}p}(s)$.

Fig.5

Extrapolated $\sigma(s)$ and $\rho(s)$ at high energies for the three analytical fits of Fig.3 (pp and $\bar{p}p$ are undiscernable). The cosmic ray data are from [17].









